ABSTRACT

Transient pressure variations within a reservoir can be treated as a propagating front and analyzed using an asymptotic formulation. From this perspective, one can define a pressure “arrival time” and formulate solutions along trajectories, in the manner of ray theory. We combine this methodology and a technique for mapping overburden deformation into reservoir volume change as a means to estimate reservoir flow properties, such as permeability. Given the entire “travel time” or phase field obtained from the deformation data, we can construct the trajectories directly, thereby linearizing the inverse problem. A numerical study indicates that, using this approach, we can infer large-scale variations in flow properties. In an application to Interferometric Synthetic Aperture Radar (InSAR) observations associated with a CO₂ injection at the Krechba field, Algeria, we image pressure propagation to the northwest. An inversion for flow properties indicates a linear trend of high permeability. The high permeability correlates with a northwest trending fault on the flank of the anticline that defines the field.

INTRODUCTION

Knowledge of the structures controlling fluid flow at depth is critical in many activities, such as petroleum production, geothermal energy extraction, CO₂ disposal, and groundwater utilization. Significant effort has been expended to increase understanding of fluid flow in the subsurface using flow and transport, and time-lapse, geophysical observations. For example, new instrumentation and reliable sensors have advanced the state of the art in flow and transport monitoring, allowing for crosswell pressure tomography, multilevel samplers, and long-term downhole pressure measurements (Freyberg, 1986; Butler et al., 1999; Hsieh et al., 1985; Pailet, 1993; Cook, 1995; Masumoto et al., 1995; Karasaki et al., 2000; Yeh and Liu, 2000; Vesseliov et al., 2001; Vasco and Karasaki, 2001; Gringarten et al., 2003).

There have been numerous efforts to use geophysical observations, particularly time-lapse data, to monitor fluid flow at depth (Tura and Lumley, 1999; Landro, 2001). Much of this work has concentrated on seismic time-lapse, or four-dimensional (4D), amplitude data from oil and gas reservoirs (Barr, 1973; Greaves and Fulp, 1987; Nur, 1989; Eastwood et al., 1994; Lee et al., 1995; Lazaratos and Marion, 1997; Huang et al., 1998; Johnston et al., 1998; Burkhardt et al., 2000; Behrens et al., 2002; Arts et al., 2004). Improved understanding of fluid flow in the shallow subsurface has resulted from shallow and crosswell time-lapse electromagnetic surveys (Brewster and Annan, 1994; Wilt et al., 1995; Barker and Moore, 1998; Binley et al., 2001; Hoversten et al., 2003).

Geodetic data provide still another set of observations related to fluid flow in the subsurface (Castle et al., 1969; Evans et al., 1982; Vasco et al., 1988; Palmer, 1990; Dusseault et al., 1993; Bruno and Bilak, 1994; Chilingarian et al., 1994; Castillo et al., 1997; Massonnet et al., 1997; Vasco et al., 1998; Wright, 1998; Wright et al., 1998; Mossop and Segall, 1999; Stancliffe and van der Kooij, 2001; Vasco et al., 2001; Du and Olson, 2001; Hoffmann et al., 2001; Vasco and Ferretti, 2005). Furthermore, current and planned instrumentation, such as permanent seafloor sensors and high-precision pressure gauges (Sasagawa et al., 2003) can generate useful displacement measurements. In addition, developments such as side-scan-sonar interferometry (Chang et al., 2000) and acoustic positioning (Spiess et al., 1998) might provide useful data.

In spite of these efforts, fundamental understanding of fluid flow within the earth is hampered by some factors. For example, there is limited data coverage, particularly with regard to flow and transport measurements. Similarly, the temporal sampling can be sparse, particularly with regard to time-lapse geophysics surveys, which can be years apart and missing important transient phenomena. Frequently, the scale of investigation is small, of the order of a few meters; and the results might not scale up into the kilometer range, in which faults and fracture zones control flow. In addition, it...
often is difficult to relate geophysical attributes, such as seismic amplitudes, to flow properties. There can be trade-offs between various sets of attributes, such as between mechanical properties and fluid pressure estimates. Finally, in applications, one typically requires knowledge of factors influencing subsurface flow in a timely fashion to optimize or mitigate the activities.

In this study, we use a cost-effective technique for imaging fluid flow and estimating flow properties, such as permeability. We use time-varying measurements of deformation in the overburden as data. Such observations are of increasing importance for at least two reasons.

First, several developments in the past few years advanced the state of the art in geodetic monitoring. For example, downhole tilt meters can provide dense time sampling of deformation at a wide range of depths, from the surface to as much as several thousand meters (Wright, 1998; Wright et al., 1998; Du et al., 2005). Space-based techniques, such as Interferometric Synthetic Aperture Radar (InSAR), can sample surface deformation with a spatial resolution (pixel size) of meters through tens of meters and with an accuracy of the order of a centimeter as to as much as a few millimeters (Massonnet and Feigl, 1998; Burgmann et al., 2000; Ferretti et al., 2000, 2001). InSAR observations might be used to image fluid-induced deformation over regions tens to as many as hundreds of square kilometers in extent (Fielding et al., 1998; Galloway et al., 1998; Amelung et al., 1999; Hoffmann et al., 2001; Schmidt and Burgmann, 2003; Colsanti et al., 2003; Vasco and Ferretti, 2005).

Second, recent developments in the processing of time-lapse seismic data allow for estimates of deformation in the overburden and within the reservoir (Guibbot and Smith, 2002; Landro and Stammmeijer, 2004; Tura et al., 2005; Hatchell and Bourne, 2005; Barkved and Kristiansen, 2005; Hall, 2006; Roste et al., 2006; Rickett et al., 2007; Staples et al., 2007; Hawkins et al., 2007). Such estimates offer high-resolution, three-dimensional sampling of deformation over a producing reservoir. Based upon a geomechanical model the deformation can be mapped into pressure changes within the reservoir (Vasco, Karasaki, and Doughty, 2000; Du et al., 2005; Vasco and Ferretti, 2005; Hodgson et al., 2007).

It is possible to use estimated pressure changes directly to infer flow properties, such as permeability (Vasco et al., 2001; Vasco, 2004b; Vasco and Ferretti, 2005). Here we take an alternative approach on the basis of the idea of a diffusive traveltine associated with the propagating pressure transient (Vasco, Karasaki, and Keers, 2000; Vasco, 2004a). Although the approach requires adequate sampling of the pressure field in time, it is relatively insensitive to the geomechanical properties within the reservoir and their heterogeneity. Furthermore, the approach shares the robustness of arrival-time tomography and actually leads to a linear inverse problem for flow properties. Thus, it is possible to map deformation measurements into reservoir permeability in a linear fashion.

**METHODOLOGY**

Before we discuss the approach in detail, we outline the major steps of our inversion methodology. The first step entails inverting measurements of overburden deformation to estimate volume change within the reservoir. Next the volume change is mapped into fluid-pressure change induced by production or injection. Finally, the fluid-pressure changes \( p(x,t) \) are used to estimate flow properties in the reservoir using a technique similar to traveltine tomography applied to a disturbance, which evolves according to the diffusion equation. Because this latter step is unfamiliar to most readers, we discuss it in more detail in this section. The first two steps are described in earlier papers (Vasco et al., 1988; Vasco et al., 1998; Mosspop and Segall, 1999; Vasco, Karasaki, and Doughty, 2000; Du and Olson, 2001; Vasco et al., 2001; Vasco et al., 2002; Vasco and Ferretti, 2005). Therefore we outline them only briefly in Appendix A.

**A diffusion equation for fluid-pressure variations**

We are interested in the propagation of pressure changes within a reservoir. The pressure changes are caused by production and/or injection from wells located within the reservoir itself. The fluid-volume changes are determined by the flow rate out of, or into, the reservoir, denoted here by the source-time function \( q(x,t) \). The reservoir will deform because of the fluid-volume changes, and that deformation will lead to displacements in the overlying material.

Our starting point is the set of partial differential equations describing the flow of an aqueous phase and a nonaqueous phase (Peaceman, 1977; de Marsily, 1986).

\[
\nabla \cdot \left( \frac{\rho_w K(x)k_{rw}}{\mu_w} \nabla (p_w(x,t) - p_w g z) \right) = \frac{\partial (\phi p_w S_w)}{\partial t} + q,
\]

\[
\nabla \cdot \left( \frac{\rho_n K(x)k_{rn}}{\mu_n} \nabla (p_n(x,t) - p_n g z) \right) = \frac{\partial (\phi p_n S_n)}{\partial t} + q,
\]

(1)

where \( S_w \) and \( S_n \) denote saturations of the aqueous and nonaqueous phases, respectively; and \( q \) denotes the injection rate of the nonaqueous phase. The relative permeabilities of the aqueous and nonaqueous phases are represented by \( k_{rw} \) and \( k_{rn} \), and the absolute permeability is given by \( K(x) \). The respective densities are \( \rho_w \) and \( \rho_n \), the gravitational constant is \( g \), and the porosity is \( \phi(x) \). The pressure associated with the aqueous phase is \( p_w(x,t) \), and the pressure for the nonaqueous phase is \( p_n(x,t) \); the respective viscosities are \( \mu_w \) and \( \mu_n \). The above equations are coupled because the saturations are assumed to sum to unity

\[
S_w + S_n = 1.
\]

(2)

Typically, in modeling \( CO_2 \)-brine systems, it is assumed that capillary effects are negligible (Pruess, 2004; Obi and Blunt, 2006) and hence \( p_w = p_n = p \), where \( p \) denotes the common fluid pressure. Adding the two equations in 1 together and assuming that the pressure in the aqueous and nonaqueous phases are equal results in

\[
\nabla \cdot [K(\kappa \nabla p - g_f) ] = \frac{\partial (\phi p \sigma_f)}{\partial t} + q,
\]

(3)

\[
\kappa = \frac{\rho_w k_{rw}}{\mu_w} + \frac{\rho_n k_{rn}}{\mu_n},
\]

(4)

\[
g_f = \frac{\rho_w k_{rw}}{\mu_w} \nabla (p_w g z) + \frac{\rho_n k_{rn}}{\mu_n} \nabla (p_n g z),
\]

(5)

and

\[
\sigma_f = \rho_w S_w + \rho_n S_n = \rho_w + (\rho_n - \rho_w) S_n,
\]

(6)

where we have used the fact that the saturations sum to unity, equation 2. Equation 3 is a differential equation describing the evolution...
of the total fluid pressure in a heterogeneous medium resulting from a time-varying source $q(x,t)$ (Rice and Cleary, 1976).

Equation 3 is a single equation in several unknowns ($p$, $S_n$, $S_w$, $\phi$, and $p_o$), and further assumptions are necessary to produce a useful expression. For example, we expand the time derivative in equation 3; thus

$$\frac{\partial (\phi \sigma_f)}{\partial t} = \sigma_f \frac{\partial \phi}{\partial t} + \phi \frac{\partial \sigma_f}{\partial t}. \tag{7}$$

Using the relationship in equation 6, we can write the second partial derivative on the right-hand side of equation 7 as

$$\frac{\partial \sigma_f}{\partial t} = \frac{\partial p_o}{\partial t} + \frac{\partial (\Delta p S_n)}{\partial t}, \tag{8}$$

where $\Delta p = p_o - p$, is the density difference between water and CO$_2$ under reservoir conditions. Typically, one can neglect the compressibility of water and treat the water density as a constant. The second term can be important when the compressibility of the nonaqueous phase is significant. For this reason, we maintain this term in our formulation.

As will be seen, because we are interested in the “arrival time” of the initial pressure change, this term will vanish because $S_n$ is insignificant at this time. Physically, this results from the fact that the pressure change propagates much faster than the saturation change. Furthermore, at the pressures within the Krenchba reservoir, the density of the CO$_2$ does not vary significantly and is close to that of water, between 0.85 and 0.90 gm/cm$^3$.

When the solid constituents are incompressible, the compressibility of the rock is dominated by changes in pore volume; and one typically has a relationship between fluid pressure $p$, the bulk or overburden pressure $p_o$, and porosity $\phi$, of the form

$$p = F(p_o, \phi). \tag{9}$$

The function $F$ usually is derived from laboratory measurements. In the fully general situation, porosity, permeability, and fluid density might be functions of fluid pressure and the overburden or confining pressure, leading to a nonlinear generalization of the diffusion equation 3 (Audet and Fowler, 1992; Wu and Pruess, 2000).

In this study, we adopt the theory of poroelasticity (Biot, 1941), in which the relationship between the fluid pressure, overburden pressure, and change in porosity is linear; thus

$$\phi - \phi_0 = \frac{\alpha}{K_u} p_o + C_e p, \tag{10}$$

which involves the initial porosity $\phi_0$, the undrained bulk modulus $K_u$, and the constants $\alpha$ and $C_e$ defined by Biot (1941). The variable $C_e(x)$ relates fluid-pressure variations to changes in porosity.

Differentiating equation 10 with respect to time gives

$$\frac{\partial \phi}{\partial t} = C_e \frac{\partial p}{\partial t} + \frac{\alpha}{K_u} \frac{\partial p_o}{\partial t}, \tag{11}$$

assuming that $\phi_0$, $C_e$, $\alpha$, and $K_u$ are time invariant. Substituting this relationship into equation 3, and accounting for the time derivatives 7 and 8, results in

$$\sigma_f C_e \frac{\partial p}{\partial t} - \nabla \cdot [\hat{K} \nabla p] + R_{e} p + \hat{q} = 0, \tag{12}$$

where $\hat{q}$ is given by

$$\hat{q} = q + \frac{\sigma_f \alpha}{K_u} \frac{\partial p_o}{\partial t} + \left( \phi_0 + \frac{\alpha}{K_u} p_o \right) \frac{\partial (\Delta p S_n)}{\partial t}, \tag{13}$$

$$R_{e} = C_e \frac{\partial (\Delta p S_n)}{\partial t}, \tag{14}$$

and $\hat{K}$ is the effective permeability

$$\hat{K} = \kappa K. \tag{15}$$

In the presence of two-phase effects, the coefficients $\sigma_f$, $\alpha$, and $\kappa$ will change with time. However, as shown next, in this application we are interested in the arrival time of a pressure disturbance as it propagates from a well. This transient pressure variation generally will propagate significantly faster than the two-phase saturation front. For this reason, we neglect the two-phase changes in $\sigma_f$, $\alpha$, and $\kappa$ and treat them as constants. Then we can apply the Fourier transform to equation 12, resulting in the frequency-domain equivalent,

$$\nabla^2 \hat{P} + \Lambda \cdot \nabla \hat{P} - i \omega D \hat{P} = \hat{Q}, \tag{16}$$

where $P(x, \omega)$ is the Fourier transform of the pressure; $\Lambda$ is the gradient of the logarithm of conductivity

$$\Lambda(x) = \nabla \ln \hat{K}(x), \tag{17}$$

which vanishes for constant $\hat{K}(x)$;

$$D(x) = \frac{\sigma_f C_e(x)}{\hat{K}(x)}, \tag{18}$$

is the reciprocal of the diffusivity; and $\hat{Q}$ is the Fourier transform of the normalized source term

$$\hat{Q}(x, \omega) = \frac{\hat{q}(x, \omega)}{\hat{K}(x, \omega)}. \tag{19}$$

If we include two-phase effects, equation 16 will contain convolutions because the coefficients $\sigma_f$, $\hat{K}$, and $R_e$ in equation 12 are then time-dependent.

An asymptotic solution to the diffusion equation

It is not possible to derive an analytic solution of equation 16 for a general variation in $\hat{K}(x)$ and $C_e(x)$. Now, numerical approaches are the most common methods for approximating solutions in arbitrary earth models. Such numerical approaches are general but do not provide much insight into the nature of pressure propagation within a heterogeneous porous medium. Furthermore, as shown below, a semi-analytic solution has some advantages in treating the inverse problem, a central concern of this work. Thus, in this subsection, we highlight a semi-analytic solution to the diffusion equation, following upon the earlier work of Cohen and Lewis (1967); Virieux et al. (1994); Vasco, Karasani, and Keers (2000); and Shapiro et al. (2002).
The approach, which is implemented in the frequency domain, is based on the notion of a high-frequency asymptotic solution, similar to methods used in the study of electromagnetic or elastic wave propagation (Friedlander and Keller, 1955; Luneburg, 1966; Kline and Kay, 1979; Aki and Richards, 1980; Kravtsov and Orlov, 1990; Keller and Lewis, 1995). However, the definition of “high-frequency” is relative to spatial and temporal variations in the background reservoir pressure in its undisturbed state, before injection or production. In other words, though the transient pressure disturbance might take days or months to propagate away from a producing well, we can consider it as a high-frequency variation when compared with the background reservoir pressure, which can vary over years, decades, or even longer. Naturally, if there are competing variations over the timescale in question, such as interference with nearby wells, we must account for that as a secondary source.

As noted by Virieux et al. (1994), an asymptotic solution to equation 16 is a power series in $1/\sqrt{\omega}$,

$$P(x, \omega) = e^{-i\omega/\sqrt{\omega} \sigma(x)} \sum_{n=0}^{\infty} A_n(x) \left( \sqrt{\omega} i \right)^n.$$

This form of the series might be deduced on physical grounds by considering a large argument expansion of the modified Bessel function of zeroth order, the solution to the diffusion equation for a homogeneous medium (Virieux et al., 1994). The motivation for a series in powers of $1/\sqrt{\omega}$ is similar to that for asymptotic methods used in modeling hyperbolic wave propagation.

In particular, when $\omega$ is large, the first term in the series (expression 20)

$$P(x, \omega) = A_0(x) e^{-i\omega/\sqrt{\omega} \sigma(x)},$$

or its time-domain equivalent obtained by inverse Fourier transforming equation 21,

$$p(x, t) = A_0(x) \frac{\sigma(x)}{2\pi t^3} e^{-\sigma^2(x)/4t},$$

(Virieux et al., 1994; Vasco, Karasaki, and Keers, 2000), will accurately represent the solution to the diffusion equation. The expression is similar in form to the solution of the diffusion equation in a homogeneous medium (Carslaw and Jaeger, 1959), but with spatially varying coefficients $A_n(x)$ and $\sigma(x)$.

To obtain explicit expressions for $\sigma(x)$ and $A_n(x)$, which are required to fully specify the solution, the sum 20 is substituted into equation 16. The operator in equation 16 might be applied term by term to the series. Substitution of the expansion 20 into equation 16 produces an expression containing an infinite number of terms. Each term will contain $1/\sqrt{\omega}$ to some power, and we could consider sets of terms for any given order. As shown in Virieux et al. (1994) and Vasco, Karasaki, and Keers (2000), the functions $\sigma(x)$ and $A_n(x)$ are determined by the terms of order $(1/\sqrt{\omega})^{-2}$ and $(1/\sqrt{\omega})^{-1}$, respectively. Because we are interested only in the phase, $\sigma(x)$, we consider only terms of order $(1/\sqrt{\omega})^{-2} \approx \omega$ in the subsection that follows.

We note that the expressions 21 and 22 are point-source solutions for a source that acts at a single location in space and at a single time. In applications, such as the synthetic and field examples discussed below, we use a step-function source, which is a more realistic representation of fluid flow rates. Such sources can be obtained by convolution with the point-source solution. In fact, any physical source can be modeled by convolution with a known source-time function. Alternatively, one could deconvolve the known source-time function from the observations and construct the equivalent point-source response. In applications below, in which the source is approximated by a step function, we simply work with the time derivative of the pressure. By linearity, this is equivalent to the pressure values resulting from a delta-function source.

There is an additional issue related to presence of the overburden pressure term in the source function $q(x, t)$ (equation 13). This term is not localized in space and could act over an interval of time. In the applications below, we assume that the overburden stress does not change significantly with time; and thus the derivative in equation 13 vanishes. However, as shown in Vasco (2004a), it is possible to correct for the overburden pressure changes, allowing for the use of the point-source expressions 21 and 22. The corrections provide a quantitative measure of the importance of variations in overburden stress in the changes of reservoir fluid pressure.

The eikonal equation, trajectories, and arrival time of the maximum pressure change

If we consider terms of order $(1/\sqrt{\omega})^{-1} \approx \omega$, we arrive at a scalar, nonlinear, partial differential equation for the phase $\sigma(x)$,

$$\nabla \sigma(x) \cdot \nabla \sigma(x) - D(x) = 0 \tag{23}$$

(Virieux et al., 1994; Vasco, Karasaki, and Keers, 2000). Equation 23, known as the eikonal equation, governs many types of propagation processes (Kline and Kay, 1979; Kravtsov and Orlov, 1990); and there are efficient numerical methods for its solutions (Sethian, 1996). The eikonal equation relates the phase function $\sigma(x)$ to the flow properties $C(x)$ and $K(x)$, as contained in $D(x)$ (equation 18).

We also might solve equation 23 using the method of characteristics (Courant and Hilbert, 1962). In the method of characteristics, solutions are developed along particular trajectories, which are denoted by $X(l)$, where $l$ signifies position along the curve. Equations for the characteristic curves are a set of ordinary differential equations, which depend on $\nabla D$:

$$\frac{dX}{dl} = s \tag{24}$$

$$\frac{d\sigma}{dl} = \nabla D, \tag{25}$$

where $s = \nabla \sigma$ (Courant and Hilbert, 1962). From equation 25, we can write the phase function as an integral,

$$\sigma(X) = -\int_{X(l)} \nabla D dl, \tag{26}$$

where $X(l)$ is the trajectory from the injection well to the observation point $x$.

The trajectory is found by solving equation 24 using a numerical technique. The method for determining the trajectory $X(l)$ from a source point to a given observation point is known as the shooting method for a two-point boundary-value problem (Press et al., 1992). Alternatively, we can determine the phase, and the trajectory, by simply postprocessing the output of a numerical reservoir simulator. This approach, which eliminates the need for ray tracing, is discussed in more detail in Vasco and Finsterle (2004) and Vasco (2004a). It makes use of the fact that
\[ \sigma = \sqrt{6T_{\max}}, \]  

where \( T_{\text{max}} \) is the time at which the pressure is a maximum. The relationship 27 is obtained by differentiating equation 22 with respect to time and setting the resulting expression to zero.

One can postprocess the results of a reservoir simulation to extract the arrival time of the peak of the pressure curve and hence \( \sigma(x) \), using equation 27. The trajectories are found by integrating the differential equation 24 using a second-order Runge-Kutta technique (Press et al., 1992). This is a much simpler procedure than ray tracing and essentially involves marching up the gradient of the phase field obtained from the simulator output (Vasco and Finsterle, 2004).

A linear inversion algorithm for flow properties

The ultimate goal is to estimate flow properties on the basis of a set of measurements of deformation of the overburden. As discussed in Appendix A, we use deformation data to estimate pressure variation within the reservoir induced by fluid injection or withdrawal. This procedure is illustrated by the synthetic and field examples below. For now, assume that we have estimates of pressure changes within the reservoir for a series of time intervals. In other words, we have snapshots of the pressure changes for a sequence of times. If the time sampling is dense enough, we can estimate the time at which the pressure change reaches a peak for points throughout the reservoir. Thus, using equation 27, we can estimate \( \sigma(x) \), for \( x \) within a region of the reservoir.

As an example, let us denote the phase estimate at point \( x_i \) by the quantity \( \Sigma_i \), and the integral by

\[ \Sigma_i = -\int_{x_i} \sqrt{D} \, dt, \]  

where \( X \) denotes the trajectory associated with the \( \Sigma_i \) observation point. Given estimates of \( \Sigma_i \) at points throughout the reservoir, we can use equation 28 to infer \( D(x) \) and consequently the flow properties that define this quantity (see equation 18).

For a dense sampling of points, we can calculate the trajectories directly from the estimates of phase, using equation 24,

\[ \frac{dX}{dt} = \nabla \Sigma. \]  

This, in effect, linearizes the inverse problem for \( \sqrt{D} \) as given by equation 28; because in estimating \( X \), directly from the phase values, we have removed the dependence of the trajectory on the unknown distribution of \( D \). Another way to see this is to return to the eikonal equation 23 and substitute in the estimate \( \Sigma(x) \) for the phase field. The result of

\[ \nabla \Sigma(x) \cdot \nabla \Sigma(x) = D(x) \]  

is a linear equation that can be solved directly for \( D(x) \). The linearity follows from the fact that we can estimate the phase field directly and do not consider it to be an unknown quantity.

By subdividing the earth model into a set of grid blocks, we can discretize the integral 28. Given a set of phase estimates derived from field data, a set of data constraint equations will result. The resulting equations for the flow properties might be written in the form of

\[ \Sigma = Gy, \]  

where \( \Sigma \) is a vector of phase estimates obtained from the deformation data; \( G \) is a coefficient matrix, which contains the trajectory lengths in each grid block of the reservoir model; and \( y \) is the vector of flow property model parameters. They will consist of estimates of \( \sqrt{D} \) in each grid block. If it is possible to estimate \( C_i \) for the region of interest, one then can solve for the effective permeability in each cell of the reservoir model.

Because the phase estimates \( \Sigma \) and the matrix coefficients of \( G \) contain errors, we do not solve equation 31 directly. Rather, we use the method of least squares to estimate the model parameters, the components of the vector \( y \) (Lawson and Hanson, 1974), by minimizing the sum of squares of the residuals. Because the equations could be inconsistent and underdetermined, we also incorporate regularization or penalty terms into the inversion (Parker, 1994). These penalty terms consist of a norm penalty, which biases the solution toward a given initial model \( y_0 \), and a roughness penalty, which favors models that are smoothly varying. The penalty terms are quadratic forms defined over the space of model parameters.

Thus, we minimize the sum

\[ R = |\Sigma - Gy|_2^2 + W_n|y - y_0|_2^2 + W_r|Dy|_2^2, \]  

where \( |z|_2 \) signifies the \( L_2 \) norm of the vector \( z \), \( W_n \) and \( W_r \) are scalar coefficients controlling the importance of the penalty terms in relation to the importance of fitting the data, and \( D \) is a matrix that approximates a differencing operator. Because the quantity \( R \) is a quadratic function of the model parameters, the condition for it to be a minimum is a linear system of equations. We solve this system of equations using the iterative solver first proposed by Paige and Saunders (1982).

APPLICATIONS

Numerical illustration

As noted above, there are several steps in the estimation procedure, each involving some degree of approximation. For example, the phase field, \( \Sigma(x) \), is inferred from the pressure estimates, which are derived from the deformation data. To illustrate the steps in this approach, and to examine how well the technique works under favorable conditions, we consider a set of synthetic deformation values. The test case was designed to correspond roughly to the actual field case that we discuss below.

A reference permeability model, shown in Figure 1a, was used to simulate the pressure response resulting from the initiation of fluid injection. (The effective permeability estimates produced by an inversion algorithm will be shown in Figure 1b.) As in the field case, the injection started suddenly and continued for more than two years. For this test, we modeled the flow rate using a step function. The reservoir model consisted of a 20-meter-thick layer, lying at a depth of 2000 meters. The numerical reservoir simulator TOUGH2 (Pruess et al., 1999) was used to compute the evolution of pressure within the reservoir resulting from the fluid injection. The code solves the pressure equation 12 numerically, using an integral finite-differences technique (Pruess et al., 1999).
The pressure variations within the reservoir are used to compute reservoir volume change assuming a linear poroelastic constitutive model (Rice and Cleary, 1976; Segall, 1985). Thus, the volume changes \( v_i \) in the reservoir are given by

\[
v_i = \mathbf{I}^{-1} \delta \mathbf{p},
\]

where \( \delta \mathbf{p} \) are the pressure changes; and \( \mathbf{I}^{-1} \) is the inverse of the coefficient matrix \( \mathbf{I} \), which is given in Appendix A (equation A-3). In calculating the volume change within the reservoir, we can account for stress transmission within the overburden. This is done for a homogeneous poroelastic overburden using the method described in Segall (1985). More complicated models of the overburden can be treated using more sophisticated numerical codes (Vasco, Karasaki, and Doughty, 2000). However, factors such as layering do not influence significantly the modeling of deformation resulting from volume change at depth (Battaglia and Segall, 2004).

The volume change within the reservoir was used to compute the deformation within the homogeneous overburden using a generalization of Maruyama’s (1964) method (Vasco et al., 1988; Vasco, Karasaki, and Doughty, 2000). In particular, we computed the range change, the change in distance to a hypothetical point in space, as discussed in Vasco et al. (2002) and Vasco and Ferretti (2005). Range change is measured by analyzing radar reflection data from orbiting satellites and provides a very accurate measure of surface displacement at high spatial densities and over large regions of the earth (Massonnet and Feigl, 1998; Burgmann et al., 2000; Ferretti et al., 2000; Ferretti et al., 2001; Colesanti et al., 2003). For the numerical illustration, we calculated range change on a 40-by-40 grid of values at the surface for 14 irregularly spaced time intervals (24, 59, 94, 129, 164, 199, 269, 304, 409, 549, 584, 689, 724, and 864 days). The time intervals correspond to those in the actual set of range-change observations described below. The range-change values for four of the intervals are shown in Figure 2. Note the smoothly varying changes, with a peak range change of approximately 1.0 cm.

The first step in the inversion procedure involves estimating the volume change within the reservoir from the range-change observations. This step is discussed extensively in other papers (Vasco et al., 1988; Vasco et al., 1998; Mossop and Segall, 1999; Vasco, Karasaki, and Doughty, 2000; Du and Olson, 2001; Vasco et al., 2001; Vasco et al., 2002; Vasco and Ferretti, 2005) and is outlined in Appendix A. It requires the solution of a linear inverse problem (equation A-1), in which the unknown parameters are grid-block fractional volume changes, and the observations are range-change data.

A row of the coefficient matrix \( \mathbf{Y} \) relating the observations to the model parameters consists of the response of the overburden to a fractional change in the volume of a grid block. The grid blocks are identical to those used in the forward calculation, with a thickness of 20 m. The entries of \( \mathbf{Y} \) depend on the spatial distribution of elastic properties of the overburden. For this illustration, we assume a homogeneous elastic half-space, which is characterized by its Poisson’s ratio (assumed to be 0.25). Details of the computation are discussed in Vasco et al. (1988, 2002) and Vasco and Ferretti (2005).

Figure 1. (a) Spatial permeability variation used to generate a set of surface displacement values. The grayscale denotes the logarithm of permeability variations within the reservoir model. (b) Logarithm of permeability multipliers resulting from the inversion of the phase values. The permeability estimates are obtained by solving the linear system of equations defined by the necessary equations for a minimum of \( R \) in equation 32.

Figure 2. Four snapshots of range change (24, 128, 408, and 842 days) induced by pressure change within the reservoir. Filled squares indicate a decrease in range, the distance between the point on the surface and a hypothetical point in space. The reference square at the top of each part signifies a change of 1.00 cm.
Next the estimated volume changes are mapped into reservoir pressure changes. This requires estimates of mechanical properties of the reservoir rock. In particular, we require the bulk modulus, as contained in \( H^{-1} \) in equation 33, throughout the reservoir. We should note that, for the arrival-time approach taken here, we do not have to take this step. Rather, we can work directly with the fractional volume change in estimating the arrival time. As long as there is a linear relationship between the volume change and the pressure change, and the relationship does not contain time derivatives, as in a viscous medium, the arrival-time estimates will be identical. This is one advantage of working with a kinematic or traveltime approach, rather than working with amplitudes themselves.

The estimated pressure changes for four time intervals (24, 128, 408, and 842 days) are shown in Figure 3. The peak pressure change is centered on the injection well and decays rapidly with distance. The pressure change is somewhat asymmetric, reflecting the heterogeneity within the reservoir. To emphasize the propagation of the transient pressure change, we estimated the time derivative of the pressure from the sequence of 14 time intervals. The resulting time derivatives, normalized by the peak values in each grid block, are shown in Figure 4. One can observe clearly the propagation of the peak pressure derivative away from the injection well as a function of time.

From the pressure derivatives, we estimate the arrival time of the peak over the simulation grid. This means that, using the pressure time series for each grid block, we estimate the time derivative and the time at which the derivative reaches a peak value. The resulting distribution of sampled arrival times is shown in Figure 5. The distribution of traveltimes is not symmetric about the injection well, an indication of the heterogeneous propagation of the pressure disturbance. The pressure arrival times, shown in Figure 5, form the basic data for an inversion for flow properties. They are used to estimate the phase field and thus compute the elements of the vector \( \Sigma \) in equation 31. Furthermore, as noted above, the phase-field estimates might be used to define the trajectories \( X \), in the integral 28 or its discrete equivalent. The trajectories are used to calculate the matrix coefficients of \( G \) in equation 31, resulting in a linear inverse problem.

The trajectories, or raypaths, computed from the phase field also are shown in Figure 5. The regularized linear inverse problem entails minimizing the quadratic form \( R \), as defined in the expression 32. The linear system of equations, comprising the necessary equations for a minimum of \( R \), is solved using the iterative linear solver LSQR (Paige and Saunders, 1982). Note that we can recover only the flow properties in regions that contain sampled phase data, the convex hull of the points in Figure 5. We have no way of knowing how fluid pressure changes propagate outside the region containing arrival-time observations.

The effective permeability estimates produced by the inversion algorithm are shown in Figure 1b. The model contains the large-
scale features, which are present in the reference model. In particular, there are high permeabilities to the north and west of the injection well and low permeabilities to the southeast. The overall pattern is a smoothed version of the reference model (Figure 1a), reflecting the influence of the roughness and norm penalties. Because of the limited extent of the phase data, it is not possible to estimate flow properties outside a region surrounding the injection well. Stated another way, it is not possible to infer flow properties beyond the region over which the pressure changes have propagated during the observation interval (0–864 days).

Another limiting factor is the decay of pressure with distance: After the disturbance has propagated a certain distance from the well, the changes will be small, producing no observable deformation at the surface. The sampling shown in Figure 5 identifies the region over which estimates are reliable. This region corresponds roughly to the area of significant permeability deviations from the background model (Figure 1b).

The regularization penalty terms in $R$ [equation 32] will trade off with the fit to arrival times. By increasing the regularization weights $W_r$ and $W_r^*$, we can degrade the arrival-time match. Thus, after completing the inversion, it is important to examine the fit to arrival times to ensure that the times were not overfit or underfit. The match is shown in Figure 6 in terms of deviations from a reference arrival time, calculated using a homogeneous starting model. In general, the calculated deviations from reference predictions appear to match the reference deviations rather well. The inverse problem is isotropic. Breaks in the topography of the structure map of the top of the reservoir layer, as determined by seismic imaging, indicate possible faults on the flanks of the anticline, trending north-northwest.

In this section, we describe an application of the methodology to range-change data associated with CO$_2$ injection in the Krechba field, Algeria. The Krechba, Teg, and Reg are gas fields distributed along a north-northwest-trending anticline. The producing layer C10.2 is a roughly 20-meter-thick layer of quartzose, fine-grained sandstone of Lower Carboniferous age. The production zone is overlain by at least one kilometer of dark-gray to black, subfissile mudstones. Above the mudstone is another kilometer of interbedded mudstones and sandstones.

The gas produced from these fields contains a high mole fraction of CO$_2$, which must be removed and disposed of for economic and environmental reasons. The CO$_2$ gas is separated from the hydrocarbons and reinjected into the reservoir at three nearby wells, KB-501, KB-502, and KB-503. The CO$_2$ injection wells are in the same formation as the producers. The horizontal wells are located on the flanks of the anticline, at depths of roughly two kilometers. The well KB-501, which we consider in this study, lies roughly 8 km to the east-northeast of the production wells, about 100 m lower in elevation.

Mud losses during the drilling of numerous wells indicate flow into pre-existing fractures, dipping at approximately $81^\circ$, striking N43$^\circ$W. The direction is subparallel to the regional stress direction determined for the Reg and Teg fields and observed throughout Algeria. The stress directions in the gas fields are remarkably consistent, striking $315^\circ$, a sign that the horizontal stress field is highly anisotropic. Breaks in the topography of the structure map of the top of the reservoir layer, as determined by seismic imaging, indicate possible faults on the flanks of the anticline, trending north-northwest.

As part of a CO$_2$ sequestration research-and-development program, Berkeley Laboratory and Tele-Rilevamento Europa (TRE) explored the use of Interferometric Synthetic Aperture Radar (InSAR) data for monitoring CO$_2$ injections. One goal of this work was to identify flow paths at depth and geologic features controlling flow. The injection of CO$_2$ into the water column induces multiphase flow because the reservoir is initially water filled. At reservoir pressure, the CO$_2$ behaves as a supercritical fluid, with a viscosity and density somewhat different from water. The well-head injection pressure was variable; on average a pressure of 15 MPa was maintained after the start of pumping, with peaks approaching 18 MPa early in the injection and about seven months into the injection. At these pressures, supercritical CO$_2$ should have a density greater than 0.85 grams/cm$^3$.

In the region of interest, the reservoir does not vary significantly in depth, and the density differences do not impact the flow. Furthermore, the advective flow almost certainly dominates the geochemical changes at depth for the two to three years of injection that we considered. We model the CO$_2$–water system as an equivalent single phase for the pressure calculations, which should be acceptable given the reservoir pressure conditions and time interval (Kumar et al., 2005).

This study utilized satellite radar images of the European Space Agency (ESA) Envisat archive from July 12, 2003, through March 19, 2007. Two satellite tracks, number 65 and number 294, surveyed the region during the CO$_2$ injection, containing 26 and 18 images, respectively. The data were processed using a permanent scatterer technique developed by Politecnico di Milano and TRE (Ferretti et al., 2000, 2001; Colesanti et al., 2003). In this approach, stable scat-
The atmospheric and orbital biases in the signal phase are corrected for, and estimates of range change are derived from the phase shifts between pairs of back-scattered radar signals. The range change represents the change in distance from a point on the earth’s surface and the nominal satellite orbit. Thus, range change is the change in the length of a hypothetical line connecting the observation location and the sensor location. Estimates of range change are influenced by variations in atmospheric moisture in space and time, corrections for topography, and land-surface changes such as growing vegetation.

Given a sequence of images, one can estimate the average range-change velocity over a given time interval, or the average change in distance to the satellite orbit over the given time interval. Similar to differential Global Positioning System (DGPS) data, all InSAR observations actually are differential measurements with respect to a reference point, which is assumed to be motionless. For example, in Figure 7 we plot the average range velocity for track 65 and track 294, corresponding to the region around the CO₂ injector KB-501. In this image, one can see the range change resulting from uplift generated by the CO₂ injection at the well. The quality of the data for this region is high because of the surface characteristics of the region; the area is primarily rock outcrops and desert with little shifting sand. The peak range velocity in Figure 7 is greater than 5 mm/year, which is several times larger than the estimated standard deviation of 1 mm/year.

Noise in the estimates is discernible if we compare the range velocity inferred for tracks 65 and 294. Each image contains notable scatter, and there are differences between the two range-velocity estimates. It should be noted that the time span covered by the two data sets is not identical, and that the number of data available is less than typically is used in permanent scatterer InSAR (Ferretti et al., 2000, 2003). This could lead to differing atmospheric corrections resulting in a long period spatial variation between the two tracks. Furthermore, the angular view of the earth’s surface is slightly different for the two data sets. The well location is situated differently in each track. The location of KB-501 is much closer to the edge of the image in track 65.

In an effort to improve the signal-to-noise ratio, we stacked the range-change estimates for tracks 65 and 294, interpolating them onto a common time sampling. The time sampling was identical to that in the numerical study, consisting of 14 time intervals (24, 59, 94, 129, 164, 199, 269, 304, 409, 549, 584, 689, 724, and 864 days). Furthermore, we averaged the scatterers spatially, using a moving window with a radius of 10 points. The resulting stacked and averaged estimates are shown in Figure 8 for four time intervals (24, 128, 408, and 842 days). The range change clearly increases with time, to as much as a peak value of more than 1 cm, and appears to migrate to the northwest.

The range-change data then were inverted for volume change within the reservoir, using the approach described in Vasco et al. (2005), and outlined in Appendix A. In essence, the linear system A-1 is solved for the volume changes in the reservoir. The Green’s function g(x,y) used in the inversion (see equation A-2) corresponded to a homogeneous half-space. The overburden is composed of layers of mudstone and quartzose sandstone. Such layering does not influence significantly the estimates of volume change (Battaglia and Segall, 2004), and a half-space model is thought to be an acceptable approximation.

Because the reservoir itself was two kilometers deep and varied in depth only by a few tens of meters in the region of interest, we approximated it by a horizontal layer. Because we did not have any direct pressure measurements, there were only deformation data, u(t) in the inversion. Including roughness and model norm penalty terms regularized the inversion. Furthermore, volume changes were penalized on the basis of their distance from the injection well KB-501.

Figure 7. Range velocities estimated from scenes associated with two satellite passes. Dark shades indicate a range decrease resulting from uplift from CO₂ injection at depth. The surface projection of the injection well is indicated by solid lines.
This means that volume changes near well KB-501 were favored over more distant changes. Thus, only the well location contributed to the inversion result; no information about flow rates within the well was available.

The volume change then was mapped into reservoir pressure change, based on equation A-5. Laboratory tests indicated that a value of \( K_s \) of 10.0 GPa was representative of the mechanical behavior of the overburden. The inferred reservoir pressure changes for time intervals of 24, 128, 408, and 842 days are shown in Figure 9. The peak pressure change of approximately 20 MPa is compatible with measurements of reservoir pressure from a downhole sensor. The migration of pressure change to the northwest is evident from the sequence of snapshots.

From the sequence of pressure changes, we estimate the phase or traveltime of the disturbance resulting from the onset of CO2 injection. Because the flow rate was nonuniform, there were several peaks in the time series of the pressure derivative. We focused on the time associated with the first peak of pressure derivative in each grid block. A more sophisticated approach, which we might attempt in a future study, involves deconvolving the flow-rate function from the time series of each grid block to produce the response to a step function in flow rate. In Figure 10, we plot the normalized time derivative associated with the first peak after 305 days of pumping. From this figure, it is evident that the pressure variation has propagated more rapidly to the northwest. In Figure 11a, the square root of the estimated arrival time is plotted for the region surrounding the injection well KB-501.

The arrival-time values form the basic data set for the inversion for flow properties. The estimates were constructed for a 40-by-40 grid of cells defining the reservoir model. However, it is possible to estimate arrival times only for those grid blocks to which the pressure change had propagated during the observation interval. The active grid blocks are shown in Figure 11b, in which the block size is proportional to the square root of the arrival time. As noted in the methodology section, the phase field defines the trajectories \( \mathbf{X}(l) \), as evident in equation 29. The trajectories associated with the phase estimates also are shown in Figure 11b, as curves connecting the phase points to the injection well. The trajectories are used to construct coefficients of the matrix \( \mathbf{G} \), which follow from a discretization of the integral 28 to a summation. The approach is identical to that used in

**Figure 8.** Range-change estimates obtained by resampling the values for tracks 65 and 294, averaging values using a moving window, and stacking the values for the two tracks. The surface projection of the injection well is indicated by solid lines in each part.

**Figure 9.** Pressure estimates resulting from an inversion of range-change data shown in Figure 8. The grayscale indicates pressures in MPa, with dark tones indicating a pressure increase. The surface projection of the injection well is indicated by solid lines in each part.

**Figure 10.** The normalized time derivative calculated from pressure estimates. The derivative is calculated for the pressure field after 305 days of injection. The derivative values have been normalized by the peak pressure in each grid block.
traveltime tomography (Aki and Richards, 1980). However, the inverse problem is linear because we have used the phase-field estimates to calculate the trajectories directly.

A regularized inversion of the linear system, equation 31, equivalent to the necessary equations for a minimum of $R$ in equation 32, was used to estimate the permeability variations in the region surrounding the pumping well (Figure 12). The coefficients $W_n$ and $W_r$ were found by trial and error, the result of some trial inversions. The goal was to find a smooth model that fit the estimated arrival times within the expected error. The permeability multipliers produced by the inversion algorithm suggest a linear, high-permeability zone cutting across the injection well and trending northwest. This feature is compatible with the migration of deformation to the northwest, which is notable in Figure 8. The orientation of the high-permeability zone is parallel to the trend of the structural anticline that defines the field.

Furthermore, the location of the high-permeability feature correlates with a break in topography, which suggests a fault cutting the eastern side of the anticline. The dashed line in Figure 12 indicates the inferred fault. The fault could extend southward; it is difficult to constrain the trace from the seismic structure map. Thus, it seems that the flow is controlled by a fault, and the permeability model is in accordance with the geologic structure of the reservoir. As stated above, we can estimate permeability only in areas that contain arrival-time observations, indicated by the filled squares in Figure 11b.

Next we examine the fit to the arrival-time data provided by the permeability model (Figure 13). Specifically, Figure 13 displays the fit to the phase data, which were derived from the deformation range-change measurements. Thus, the “data” actually are estimates produced first by inverting the range change for pressure change and then by estimating the arrival time from the time-varying

---

**Figure 11.** (a) Arrival time of the pressure transient, calculated from pressure derivative estimates. (b) Solid squares denote phase or arrival-time estimates used in the inversion for permeability. Solid lines represent trajectories used in the inversion. The trajectories are calculated from the phase field, as indicated in equation 29.

**Figure 12.** Logarithm of permeability multipliers resulting from the inversion of phase values in Figure 11b. The surface projection of the injection well is indicated by the solid line. The surface projection of a fault, inferred from seismic data, is indicated by the dashed line.

**Figure 13.** Observed deviation of traveltimes from those predicted by a uniform permeability model, the background model. The observed deviations are plotted against deviations predicted by the inversion result. For perfect agreement, the observed and predicted deviations would plot along the 45° line shown in the figure.
pressures. Each inversion will introduce some level of approximation and bias caused by the inclusion of regularization terms and by errors in the actual measurements. Overall, the travel times predicted by the permeability model appear to agree with the observed travel times.

Finally, we consider the influence that variations in the range-change data have on permeability estimates. In particular, there are differences in the range-change velocity estimates for tracks 65 and 294, which are visible in Figure 7. To examine the stability of the permeability estimates, we conducted an inversion in which data was used only from track 65. The resulting phase estimates are shown in Part a, Figure 14a. The general features of these estimates—the asymmetric distribution of the phase, the faster propagation to the northwest, and the overall extent of the contours—agree with the averaged range-change estimates (Figure 11a). Furthermore, the phase estimates from track 65 are in qualitative agreement with estimates from track 294 (Figure 14b).

However, there are some differences in detail among the various estimates. An inversion of the phase data from tracks 65 and 294 results in the permeability multipliers shown in Figure 15. Again, the

Figure 14. (a) Phase changes associated with an inversion of range-change data from track 65. (b) Phase changes associated with an inversion of range-change data from track 294.

Figure 15. (a) Permeability deviations from the uniform background model obtained from an inversion of phase changes associated with track 65 (see Figure 14). (b) Permeability deviations from the uniform background model obtained from an inversion of phase changes associated with track 294.
general features of the permeability estimates, such as the north-west-trending linear, high-permeability anomaly and the magnitude of the deviations, are the same for the two inversions. The primary differences in the results are the southeast extent of the high-permeability trend. There also is a slight shift of the anomalies to the southwest in the inversion of data from track 294, relative to the inversion results from track 65. This shift might result from use of the same look vector for the two inversions. In reality, the satellites were in somewhat different positions for the two tracks.

CONCLUSIONS

Fluid-pressure changes within a reservoir or aquifer can produce measurable deformation within the overburden. Such deformation has been observed in both geodetic data and time-lapse seismic surveys. The resulting deformation can be used to infer pressure and pressure changes within the reservoir. Although it is possible to map the estimated pressure changes directly into flow properties, in this study we use an alternative approach on the basis of the idea of a pressure “arrival time.” The primary advantage of this approach is that it is not sensitive to mechanical properties within the reservoir. In other words, if the reservoir behaves elastically over each time increment, the arrival-time estimate can be derived from the volume change within the reservoir and does not require mapping the volume change into pressure change. Thus, we do not need to estimate the value of $K$, and its variation, within the reservoir (see equation A-6).

An additional advantage of this approach is that the phase field is estimated from the deformation data and can be used to compute the raypaths or trajectories for the tomographic traveltime inversion. This, in effect, linearizes the inverse problems; and one then has all the tools of linear inverse theory available. Thus, the estimation of flow properties, on the basis of deformation data, is a linear problem. Furthermore, one can compute useful measures for model assessment, such as the model parameter resolution matrix. We will illustrate this in a follow-up paper on the analysis of range change associated with injection into well KB-502.

Application of the technique to InSAR range-change estimates from the Krechba field in Algeria indicates that the approach is practical. The linear, high-permeability feature produced by the injection into well KB-502. The possibility exists that the fault is deforming in response to the injected fluid. This mechanism might involve changes in permeability resulting from deformation, a subject not addressed in this study.

One limitation of this approach is the need for adequate sampling of the deformation field in time and space. In the near future, a growing number of satellite platforms, mounting high-resolution radar sensors, will become available (e.g., Radarsat-2, TerraSAR-X, Cosmo-Skymed), providing additional coverage. Dense spatial sampling in three dimensions also can be obtained through the use of time-lapse seismic data, although at a higher cost. Permanent seafloor sensors could be used to improve the sampling in time. In addition, it is possible to apply the arrival-time methodology to pressure estimates obtained directly from time-lapse seismic data, bypassing entirely the estimation of deformation. An extension of this work would allow for permeability changes to accompany deformation; that is, there might be coupling between deformation and flow properties. Such coupling might occur when the $\mathrm{CO}_2$ is channeled into a narrow fault zone. We plan on considering pressure-dependent flow properties and their estimation in a future study.

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APPENDIX A

DEFORMATION IN THE ELASTIC OVERBURDEN

When porosity changes occur, induced by fluid pressure buildup or decay in a reservoir or aquifer, there is a corresponding volume change. The volume change induces deformation in the surrounding material, which is transmitted through the subsurface. In this Appendix, we note how such deformation might be used to estimate reservoir pressure changes.

Estimation of volume change

The first step in the estimation procedure is to use the deformation data to infer changes in pressure. This aspect is discussed thoroughly by Vasco et al. (2001). We state only the final result, a system of linear equations relating volume changes within each grid block of a reservoir model $v_j(t)$ to pressure changes $\delta p(t)$ and deformation observations $\mathbf{u}(t)$. Thus,

$$
\begin{bmatrix}
\mathbf{u}(t) \\
\delta p(t)
\end{bmatrix} =
\begin{bmatrix}
Y & II
\end{bmatrix}
\begin{bmatrix}
v_j(t)
\end{bmatrix} 
$$

(A-1)

where coefficients of the matrices $Y$ and $II$ are given by

$$
Y_{ijk} = \int_{V_j} g_i(\mathbf{x}_k, \mathbf{y}) dV, \quad (A-2)
$$

for observation point $\mathbf{x}_j$, deformation components $i$ and reservoir grid block $j$, and

$$
II_{ijk} = C_e^{-1} \left[ \delta_{jk} - \frac{B}{\rho} \int_{V_j} G(x_k, \mathbf{y}) dV \right], \quad (A-3)
$$

respectively; where $B$ is Skempton’s pore-pressure coefficient (Rice and Cleary, 1976; Wang and Kuempel, 2003), $\rho$ is the density, and there is no summation over the repeated index $i$.

The integrand in expression A-2, $g_i(\mathbf{x}_k, \mathbf{y})$, is the Green’s function, which is the point response of the elastic medium at location $\mathbf{x}_k$. 

Therefore, the matrix $Y$ defines the deformation produced by unit pressure changes at each of the reservoir observation points, and the matrix $II$ defines the change in pressure produced by a unit deformation at each of the observation points.
Therefore, it is medium dependent, as discussed in Vasco, Karasaki, and Doughty (2000). The integral is evaluated over the volume of the \( h \)-th grid block, \( V \). The kernel \( G_0 \) in the integral in expression A-3 is constructed from the Green’s function solution \( g \), as described in Segall (1985).

\[
G_0 = (2\mu + \lambda) \frac{\partial^2 g}{\partial x_i^2}, \quad (A-4)
\]

Note that one could allow the poroelastic coefficients \( C_{ij} \) to vary over the aquifer or reservoir, assuming that such information is known. The system of equations A-1 typically is unstable with respect to perturbations, numerical or otherwise; and some form of regularization is required. Thus, the system is solved using a regularized least-squares algorithm, as discussed in Vasco, Karasaki, and Keers (2000) and Vasco et al. (2001).

Mapping estimates of volume change into changes in pressure and overburden stress

Once estimates of the volume change within the reservoir are available, by solving the system of equations A-1, we map them into fluid pressure and overburden stress changes. The mapping is described in more detail in Vasco et al. (2001) and is based on the work of Segall (1985). The fluid pressure change is given by

\[
\delta p(x,t) = C_v v(x,t) - \frac{B^2}{3\rho} \sum_{i=1} G_0(x,y) v_i(y,t) dV, \quad (A-5)
\]

where

\[
C_v = \frac{B^2}{\rho} \left[ \frac{2(1 + \nu_d)(1 + \nu_u)}{\nu_d - \nu_u} - 3K_v \right], \quad (A-6)
\]

\( \nu_d \) and \( \nu_u \) are the undrained and drained Poisson’s ratios (Rice and Cleary, 1976), \( K_v \) is the undrained bulk modulus, and the diagonal components of the overburden stress are given by

\[
\sigma_{ii} = \frac{B}{\rho} \int_{V} G_0(x,y) v_i(y,t) dV - \frac{B K_u}{\rho} v_i(x,t), \quad (A-7)
\]

Note that all mappings are linear, as is the inverse problem for volume change, equation A-1. The second term in equation A-5 accounts for propagation of stress within the overburden in the calculation of reservoir pressure change.

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