Diameters of the Orbital Tubes in Long-Term Interferometric SAR Surveys

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***Abstract*—This letter studies the impact of the use of permanent scatterers (PS) on the distribution of the perpendicular baselines in long-term satellite interferometric synthetic aperture radar surveys. This letter also evaluates the relation between the radar center frequency and the dispersion of the estimates of the elevations of the PS as a function of noise and of the time jitter due to atmospheric disturbances.**

***Index Terms*—Diameters of the orbital tubes, differential interferometry, differential synthetic aperture radar interferometry (DINSAR), permanent scatterers.**

I. Introduction

THE USE OF satellite-mounted radars for synthetic aperture radar interferometry (INSAR) and differential synthetic aperture radar interferometry (DINSAR) has been demonstrated and is now operational with several platforms (European Remote Sensing satellites 1 and 2 (ERS 1/2), the Environmental Satellite (ENVISAT), Radarsat, Japanese Earth Resourses Satellite (JERS-1), and several others in the near future). In this letter, I shall analyze the impact of the use of stable scatterers [the so-called permanent scatterers (PS)] on: 1) digital elevation models (DEMs) determined using INSAR; and on 2) DINSAR. The intent is to prove that the optimal management of the orbital tube of the satellite (i.e., the allowed perpendicular baselines) does not correspond to make it as narrow as possible. Similar to what happens for the parallax in aerophotogrammetry, it is useful to achieve, at least for the initial (calibration) phase of the survey, a wide dispersion of the baselines, as allowed by the limitedly pointwise behavior of the scatterers. Thereafter, for most of the time, the baselines should be kept small.

Another result of this analysis is the evaluation of the effects of the center radar frequency in connection with the time jitter induced by atmospheric disturbances and the limited SNR available.

The basic ideas are the following. First, the dispersion of the elevation of the stable scatterers is determined as a function of the number of takes and of the dispersion of the perpendicular baselines; the noise is supposed to be mostly due to atmospheric time jitter. Obviously, the more dispersed are the baselines, the better is the estimate of the elevations. Then, if the elevations of the permanent scatterers are well determined, one can obtain a good estimate of the atmospheric phase screen (APS), in correspondence of these points, since the phase contribution measured is due to elevation (known rather well in that point) and to the atmosphere. Thus, the APS estimate improves with the dispersion of the baselines. As a consequence, the quality of the motion estimation improves with the quality of the APS estimation.

II. Phase Dispersion of the Permanent Scatterers

In order to evaluate the advantages of surveys repeated for long time intervals, I suppose to have a set of points (the permanent scatterers) that have a radar signature unchanged with time [3], [4]. The measured phase for the jth point at the ith passage is indicated as

$$\phi_{ij} = \beta_i \frac{q_j}{D} + \alpha_{ij}, \qquad i = 1:N$$

$$\beta_i = \frac{B_i}{B_{\text{cr}}} \quad B_{\text{cr}} = \frac{RB_{\text{RF}}}{f_0}\tan\theta \tag{1}$$

$$q_j = \frac{\lambda R \sin\theta}{4\pi B_{\text{cr}}} \frac{(\phi_{ij} - \alpha_{ij})}{\beta_i} = D\frac{(\phi_{ij} - \alpha_{ij})}{\beta_i} \tag{2}$$

$$D = \frac{c\cos\theta}{4\pi B_{\text{RF}}} \sim 1.42 \text{ m (in the case of ERS)} \tag{3}$$

where q_j is the elevation, B_i the baseline at the ith passage measured normally to the radar line of sight, B_{cr} the critical baseline [2], f_0 the central frequency, B_{RF} the radio-frequency bandwidth, and α_{ij} the atmospheric contribution, hereon referred to as APS. Its variance σ_α^2 is dependent on the variability of the two-way delay of the electromagnetic waves in the atmosphere, mostly due to water vapor [5]. Other elements can influence the phase, such as uncertainty of the baselines, etc. In this letter, I will concentrate on the atmospheric factor, in order not to increase the complexity too much. The variance of the delay is given in terms of the variance of the additional two-way travel path σ_m^2, approximately wavelength independent, and posed in this letter to be equal to about 1 cm^2, to give an idea of the orders of magnitude of the entities involved. This is, in practice, a time jitter, and its phase grows with the frequency. Moreover, additive phase noise, be it due to clutter, electronic noise, ambiguities, etc., contributes another σ_n^2. This contribution is in general negligible with respect to that due to the atmospheric effects, but it will also be considered, since it may become significant for lower frequencies. The formula for the phase variance indicated in [7] is not used, since the scatterer is deterministic and not stochastic. Rather, a factor 2 is considered, to refer to the in phase component of the noise only

$$\sigma_\alpha^2 = \sigma_m^2\left(\frac{2\pi}{\lambda}\right)^2 + \sigma_n^2 = \sigma_m^2\left(\frac{2\pi}{\lambda}\right)^2 + \frac{1}{2 * \text{SNR}}.$$

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Digital Object Identifier 10.1109/LGRS.2004.830125

Now, if after N observations, we optimize $\widehat{q}_j$ the estimate of q_j, the elevation estimation error is, for the jth PS

$$\varepsilon_{qj} = q_j - \widehat{q}_j.$$

Then, the estimate of the APS, $\widehat{\alpha}_{ij}$ will be

$$\widehat{\alpha}_{ij} = \phi_{ij} - \beta_i \frac{\widehat{q}_j}{D}$$

and the error in the APS estimate in correspondence of the position of the permanent scatterer is

$$\varepsilon_{\alpha ij} = (\widehat{q}_j - q_j) \frac{\beta_i}{D}$$

and grows with the baseline. Now, it can be seen (see the Appendix) that the mean square value of ε_{qj} is

$$E\left[\varepsilon_{qj}^2\right] = D^2 \frac{\sigma_\alpha^2}{N\sigma_\beta^2}.$$

In the case of ERS geometry and for an SNR greater than 1, for example (see Section V), we have

$$E\left[\varepsilon_{qj}^2\right] = \frac{1}{N} \frac{D^2}{\left(\frac{500}{1050}\right)^2} \sim \frac{8.77}{N} \text{ m}^2$$

where σ_α^2 is the variance of the APS values at 5 GHz (about 1 rad), and σ_β is the standard deviation of ERS or ENVISAT baselines (about 500 m) if normalized to the critical baseline, i.e., in this case 1050 m, i.e.,

$$\sigma_{\beta,\mathrm{ERS}}^2 = \left(\frac{500}{1050}\right)^2 \sim 0.23.$$

The variance of the errors of the atmospheric estimates, at the ith passage, in correspondence of the various PS is

$$\sigma_{\varepsilon\alpha i}^2 = E\left[\varepsilon_{\alpha ij}^2\right] = \frac{\sigma_\alpha^2}{N\sigma_\beta^2}\beta_i^2 \tag{4}$$

where β_i is again the ith baseline, normalized to the critical value. On average

$$E\left[\sigma_{\varepsilon\alpha}^2\right] = \frac{\sigma_\alpha^2}{N\sigma_\beta^2} \quad E\left[\beta_i^2\right] = \frac{\sigma_\alpha^2}{N} \tag{5}$$

so that on average the residual error in the APS estimate depends only on the number of passages N. Basically, we have a number of points that stay still; their phases change depending on the baselines, and the wider the baselines, the better is the estimate of their height. Once we have measured well their heights, then the measured phases will yield an estimate of the APS there: the better, the smaller is the baseline, since smaller will be the impact of the elevation error.

III. Discussion

On average, the quality of the atmospheric estimate and, therefore, of the measurements depends only on N. However, it is evident that the following strategy is paying: first, carry out surveys with large values of the baselines, to increase as far as possible σ_β^2 and to be able to well determine the PS elevations. After this initial phase, that could coincide with the calibration phase, low baselines are optimal in that the elevation errors obtained in the first phase will be minimally influencing the estimates of the APS, and these measurements will be optimal. As the APS is the major error source for motion estimation, all improvements in its estimate entail correspondent improvements in the estimate of motion.

This proves the necessity of long surveys where baselines, large at the beginning, are to be followed by sequences of small ones. In other words, the distribution of the baselines should be long tailed, but with a rather narrow central lobe. The extension of the longer baselines is dependent on the capability of the PS to maintain the same signature, i.e., on their pointwise character. Some work has been already carried out to analyze the maximal baseline that should be allowed, in order not to lose too many PS, but at the same time, optimizing the quality of their position [1]. It has been seen that, in the case of ENVISAT, the baselines can outgrow the critical baseline. The aforementioned strategy is better than keeping σ_β^2 as small as possible, in a "pure" differential interferometric experiment; this would lead to the result in (5). In other words, to expand σ_β^2 with "a few" wide baselines is beneficial for the entire survey. Furthermore, it would also be beneficial if the wide baseline takes are carried out at the beginning of the mission, say during the calibration phase. Thus, the advantages of an improved DEM estimate and the ensuing improvements in the APS screen estimate and then in motion estimation would be usable during all the successive operations.

IV. Interferometry at Different Frequencies

The noise and the atmospheric time jitter yield a phase shift that changes with the wavelength as follows [6]:

$$\sigma_\alpha^2 = \sigma_m^2 \left(\frac{2\pi}{\lambda}\right)^2 + \sigma_n^2 \tag{6}$$

and neglecting for now σ_n^2, as it will be justified in the following, we have:

$$\sigma_{\varepsilon\alpha i}^2 = \left(\sigma_m \frac{2\pi}{\lambda}\right)^2 \frac{1}{N\sigma_\beta^2}\beta_i^2$$
$$\sigma_{\varepsilon q}^2 = E\left[\varepsilon_{qj}^2\right] = \left(\frac{\cos\theta}{2}\frac{f_0}{B_{\mathrm{RF}}}\right)^2 \frac{\sigma_m^2}{N\sigma_\beta^2} = \left(\frac{R\sin\theta}{2B_{\mathrm{prms}}}\right)^2 \frac{\sigma_m^2}{N} \tag{7}$$

where B_{prms} is the rms value of the perpendicular baseline. In other words, the last equation tells us that if the orbital tube is kept to the minimum value correspondent to the platform positioning constraints, independent of the wavelength, $\sigma_{\varepsilon q}^2$ grows. Whereas, if the normalized baseline variance σ_β^2 is kept constant, and therefore "a few" baselines are wider and wider the higher is B_{cr}, then $\sigma_{\varepsilon q}^2$ is practically only dependent on the inverse relative bandwidth f_0/B. Hence, for a given satellite SAR, the orbital tube should not be as narrow as possible. The same considerations extend to motion estimation, because $\sigma^2_{\varepsilon\alpha i}$ decreases increasing σ^2_β. In fact, for motion estimation $\sigma^2_{\varepsilon mi}$, the variance of the noise disturbing the two-way motion estimate at the ith passage will be that due to the atmospheric time jitter *reduced by the factor* $\beta_i^2/N\sigma_\beta^2$ but independent of the center frequency

$$\sigma^2_{\varepsilon mi} = \left(\frac{\lambda}{2\pi}\right)^2 \left(\sigma_m \frac{2\pi}{\lambda}\right)^2 \frac{1}{N\sigma^2_\beta}\beta_i^2 = \sigma_m^2 \frac{\beta_i^2}{N\sigma^2_\beta}. \quad (8)$$

We see that the higher phase variance at higher frequencies impacts on the variance $\sigma^2_{\varepsilon\alpha}$ of the estimate of the APS, but not on $\sigma^2_{\varepsilon q}$, that of the elevations, mostly dependent on the square inverse of the relative bandwidth. Say, for L-band systems, we have a variance of the elevations of the PS lower than that measurable with similar X-band systems, since the ratio f_0/B_{RF} is much more favorable. Correspondingly, the B_{prms} should be higher, if the orbital tube is not kept as narrow as possible on purpose. In other words, *all this is true if* we keep σ_β high.

We are now able to evaluate the impact of reinitializing any survey, say changing the wavelength or the incident angle.

- N restarts from 0;
- we lose the initial high baseline passages.

If we refer to $N = 100$, a typical figure now for ERS archives, and we refer to monthly passages, then at least three years are necessary to lose less than 5 dB in precision, say making first six long baseline passes and then 30 low baselines ones. The precision that can be obtained is metric in elevations, and millimetric in motion retrieval.

V. Effects of Additive Noise

We discuss now the impact on DEMs estimates and on differential interferometry of possible additional noise n_{ij}, be it due to clutter, ambiguities, or else, besides that due to the atmosphere, a time jitter that scaled with frequency. We observed that the power of the in-phase noise is, for each PS

$$E\left[n_{ij}^2\right] = \frac{1}{2 \times \mathrm{SNR}}.$$

From (7), we have that for SNR high enough (see the Appendix) the elevation error is dependent on the relative bandwidth. The use of PS and the existence of long relative baselines (high σ_β) improves the estimate. However, this is true if the error is supposed to be mostly due to the atmosphere. From (6), we see that the longer the wavelength, the additive noise is negligible only if the SNR becomes higher. In order to appreciate this SNR, we impose that after averaging over the area of interest, populated by M PS, the effect of the additive noise is equal to that of the *residual atmospheric contribution*. In fact, the additive noise is independent from one PS to the next, whereas the atmospheric contribution is practically the same, at least until the radius of the area of interest exceeds that of the correlation of the atmospherical artifacts, say several hundred meters. Therefore, for any point that we wish to analyze, we estimate the local value of the residual atmospheric phase screen, and then we compare this residual to the averaged local additive noise. Thus, we have

$$\left(\sigma_m \frac{2\pi}{\lambda}\right)^2 \frac{1}{N\sigma^2_\beta}\beta_i^2 \sim \frac{1}{2M \cdot \mathrm{SNR}}$$

$$\lambda < \sigma_m \times \frac{2\pi\beta_i}{\sigma_\beta}\sqrt{\frac{2M \cdot \mathrm{SNR}}{N}}$$

$$\lambda < 0.01 \times 2\pi\sqrt{\frac{2 \times 30}{50}}\sqrt{M} = 0.068 \text{ m}.$$

Posing $\beta_i \sim \sigma_\beta$ and for a limit SNR equal to 15 dB, then $\lambda < 7$ cm for $M = 1$, i.e., if the area of interest contains just one PS. As soon as the radius of the area of interest increases beyond this minimum value, this constraint practically vanishes. Summarizing, for lower frequencies we have to consider two effects.

1) the critical baseline becomes really too wide making less likely the possibility of high values for σ_β. Then, results more similar to those indicated in (5) should be accepted.
2) the additive noise effects become relevant. If only one PS is available that has the motion to be measured, its SNR should be high. Obviously, if several PS share the same motion, the requisite on the SNR can be relaxed.

VI. Conclusion

The previous simple analysis entails interesting conclusions.

- Quality of the retrieved topography depends mostly on the relative bandwidth, but only if the relative baseline variance σ^2_β is as high as possible.
- Quality of differential interferometry (i.e., the quality of the motion estimation) is basically frequency independent (what is gained with the shorter wavelength is lost to the atmospherical noise), provided that the wavelength be shorter than, say, 7 cm. Longer wavelengths require SNR proportionally higher than 15 dB or averaging over sizeable areas.
- The technique of the permanent scatterers yields two distinct advantages. First and foremost, the selection of the stable points allows long surveys. Second, the APS estimate can be improved with respect to interferogram stacking, by exploiting the pointwise character of most PS reflectors to allow a long tailed distribution of the baselines.

These longer relative baselines allow to increase σ_β, first to improve the quality of the topography and then that of the APS estimate.

Further research should be carried out to analyze the variations of density and mechanical stability of the PS with frequency, bandwidth, polarization, etc. to complete the analysis.

Appendix

If we have measures x_i of the elevation q, with noise n_i

$$x_i = q + n_i$$

and variance

$$E\left[n_i^2\right] = \sigma_i^2 = \left(\sigma_\alpha^2 + \frac{1}{2 \times \mathrm{SNR}}\right)\left(\frac{D}{\beta_i}\right)^2$$

where σ_α^2, D are constant, the best linear estimate $\widehat{q}$ is

$$\widehat{q} = \sum \mu_i x_i \quad E\left[(q-\widehat{q})\, x_j\right] = 0 \quad E\left[x_i x_j\right] = \sigma_q^2 + \sigma_j^2 \delta_{ij}.$$

The variance of the estimate of the elevation $\sigma_{\varepsilon q}$ is

$$\mu_j = \frac{\frac{\sigma_q^2}{\sigma_j^2}}{1+\sum\limits_i \frac{\sigma_q^2}{\sigma_i^2}};\ \sigma_{\varepsilon q}^2 = \frac{\sigma_q^2}{1+\sum\limits_i \frac{\sigma_q^2}{\sigma_i^2}} = \frac{1}{\frac{1}{\sigma_q^2} + \frac{1}{D^2 \sigma_a^2} \sum\limits_i \beta_i^2}.$$

For ERS and for SNR $\rightarrow \infty$, $\sigma_q^2 = \infty$, $\sigma_\alpha^2 = 1$ we have

$$\sum_i \beta_i^2 = N\sigma_\beta^2 = 0.23N$$

$$E\left[\varepsilon_{qj,N}^2\right] = \sigma_{\varepsilon q}^2 = \frac{D^2 \sigma_a^2}{N\sigma_\beta^2} = \frac{8.77}{N}\left(\mathrm{m}^2\right).$$

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