3D motion recovery with multi-angle and/or left right interferometry

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Abstract

Interferometrical techniques yield motion along the line of sight. In order to retrieve the three motion components, at least three non complanar LOS vectors are needed. This can be achieved first with ascending and descending orbits and also with different incidence angles or with left right observations. The signal to noise ratios achievable for the different components of the motion (best up down, then EW, and worst NS) using multiple surveys are presented. The use of image differences rather than interferometry is also considered.

Introduction

The interferometrical measurements yield the displacement of the target along the Line of Sight (LOS). To measure the three components of the motion vector **x** of an image point:

$$\underline{\mathrm{x}} = [\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3]$$

we need **N>3** motion components along LOS motion vectors not belonging to the same plane. We need for instance, ascending and descending orbit data, at two different angles, or left – right acquisitions. SAR Differential Interferometry yields the components v_i of the pixel motion vector $\underline{x}$ along the i-th (*i=1…N*) Line Of Sight vector $\underline{\mathbf{a}}_\mathbf{i}$

$$v_i = \underline{a}_i^T \underline{x}$$

$$\begin{bmatrix} v_1 \\ v_1 \\ .. \\ .. \\ v_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ .. & .. & .. \\ .. & .. & .. \\ .. & .. & .. \\ a_{N1} & a_{N2} & a_{N3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\underline{\mathbf{v}} = \underline{\mathbf{A}}\underline{\mathbf{x}}$$

In order to evaluate the quality of the measurement and the possibility of retrieval of the three components, we can calculate the eigenvalues of the (3x3) square matrix,

$$\underline{\mathbf{C}} = \underline{\mathbf{A}}^T \underline{\mathbf{A}}$$

where $\underline{\mathbf{A}}$ is the (3xN) rectangular matrix of the components of all available LOS vectors. In fact, the matrix $\underline{\mathbf{C}}$ has to be inverted in order to recover, using pseudo inversion, all motion components in the least square sense. The variances of the components of the noise vector $\underline{\mathbf{n}}$ are inverse to the number of acquisitions dedicated to left or right views. Using a bayesian approach and calling ε the noise to signal ratio and also supposing ε to be very small in order to avoid bias, we have:

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$$\underline{\mathbf{v}} = \underline{\mathbf{A}}\,\underline{\mathbf{x}} + \underline{\mathbf{n}}$$

$$\mathbf{E}\left[\underline{\mathbf{n}}\,\underline{\mathbf{n}}^{\mathrm{T}}\right] = \boldsymbol{\sigma}_{\mathbf{n}}^{2}\underline{\mathbf{W}}$$

$$\mathbf{E}\left[\underline{\mathbf{x}}\,\underline{\mathbf{x}}^{\mathrm{T}}\right] = \boldsymbol{\sigma}_{\mathbf{v}}^{2}\underline{\mathbf{I}}$$

$$\boldsymbol{\varepsilon} = \frac{\boldsymbol{\sigma}_{\mathbf{n}}^{2}}{\boldsymbol{\sigma}_{\mathbf{v}}^{2}}$$

$$\mathbf{E}\left[\underline{\mathbf{v}}\,\underline{\mathbf{v}}^{\mathrm{T}}\right] = \boldsymbol{\sigma}_{\mathbf{v}}^{2}\underline{\mathbf{A}}\,\underline{\mathbf{A}}^{\mathrm{T}} + \boldsymbol{\sigma}_{\mathbf{n}}^{2}\underline{\mathbf{W}}$$

$$\underline{\hat{\mathbf{x}}} = \underline{\mathbf{B}}\,\underline{\mathbf{v}}$$

$$\left(\underline{\mathbf{B}}\,\underline{\mathbf{v}} - \underline{\mathbf{x}}\right)\underline{\mathbf{v}}^{\mathrm{T}} = \underline{\mathbf{0}}$$

$$\underline{\mathbf{B}} = \underline{\mathbf{A}}^{*}\left(\underline{\mathbf{A}}\,\underline{\mathbf{A}}^{\mathrm{T}} + \boldsymbol{\varepsilon}\underline{\mathbf{W}}\right)^{-1}$$

$$\underline{\mathbf{Q}} = E\left[\left|\underline{\mathbf{B}}\,\underline{\mathbf{v}} - \underline{\mathbf{x}}\right|^{2}\right]$$

The error matrix $\underline{\mathbf{Q}}$ has to be scaled back; the error components are along its eigenvectors

$$\boldsymbol{\varepsilon}\underline{\mathbf{Q}} = \underline{\mathbf{I}} - \underline{\mathbf{A}}^{*}\left(\underline{\mathbf{A}}\,\underline{\mathbf{A}}^{\mathrm{T}} + \boldsymbol{\varepsilon}\underline{\mathbf{W}}\right)^{-1}\underline{\mathbf{A}}$$

We consider two different scenarios: in the first, only right acquisitions are allowed, and we draw three curves (continuous, red) yielding the three eigenvalues of the (3x4x4x3) $\underline{\mathbf{C}}$ matrix correspondent to ascending and descending right acquisitions taken at 23 degrees incidence and also at a different incident angle starting from 20 to 55 degrees, as a function of the incidence of the second two acquisitions (Figure 1).

In the other scenario, eight different acquisition modes are available; namely, we add four more to the previous four, using now left and right acquisitions. Then, the eigenvalues of the (3x8x8x3) $\underline{\mathbf{C}}$ matrix are plotted on the same figure 1, using blue dots. The curves show that indeed left - right acquisitions yield identical quality as the right acquisitions only, for the motion components Up – Down (the maximal eigenvalue, approximately equals to a 6dB *SNR* gain corresponding to four acquisitions) and East-West. The North-South component, totally invisible in the first scenario if the second incidence angle is once again 23 degrees, becomes more and more visible as the second incident angle increases to about 50 degrees. In this case, the quality of just this component is about 8 dB worse than that available using left - right acquisition. Furthermore, it may be useful to remark that in the case of left-right acquisitions, the utility of different angles is not too great. In figure 2, 3 I show what would happen at latitudes other than the temperate ones. Then, in figure 4, I show what happens with a mixture of acquisitions, dedicated only partly to the left ones. The results, as expected, are intermediate.

LOS measurements should be carried out exactly for the same position, to avoid combining LOS measurements of different targets. This pushes towards the use of say corner reflectors, if high quality 3-component motion measurement is required; in that case, a CR with side 1.6 times wider would yield, for left acquisitions only, the same results found using both left and right at 50% each. In many occasions the modeling of the phenomenon may yield further information on the motion vector; the motion could be, say, down hill for landslides; along the flow, for glaciers. In the case of volcanoes inflations or deflations, however, the recovery of 3D motion would be important.

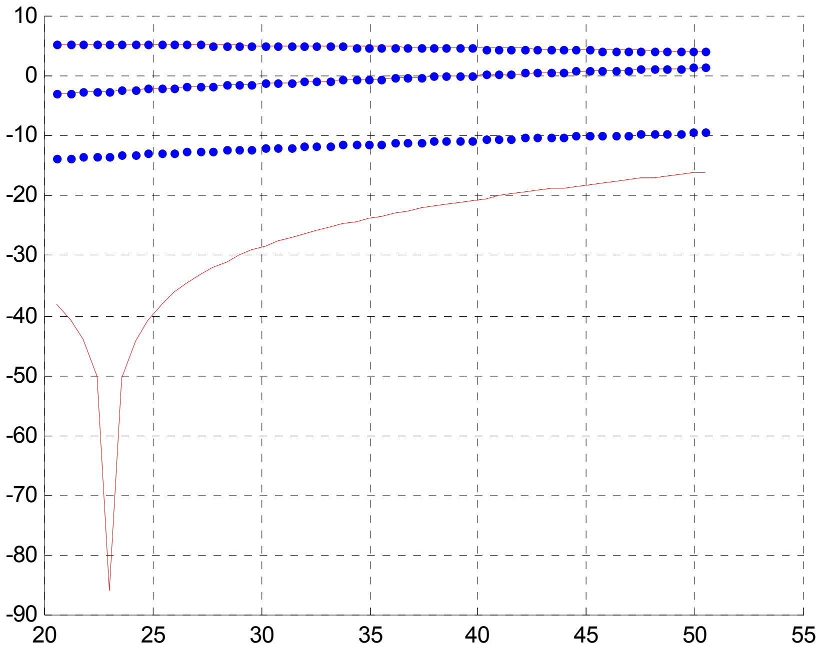

Figure 1: Signal to noise ratio gain (or loss) for different components of the motion vector as a function of the incidence angle of the second (or fourth) pair of acquisitions. The incidence angle of the first pair is 23 degrees. (Temperate latitudes). Upper curve U-D; middle curve E-W; Lower curve N-S component. Blue dots correspond to left right acquisitions; red curves to right acquisition only.

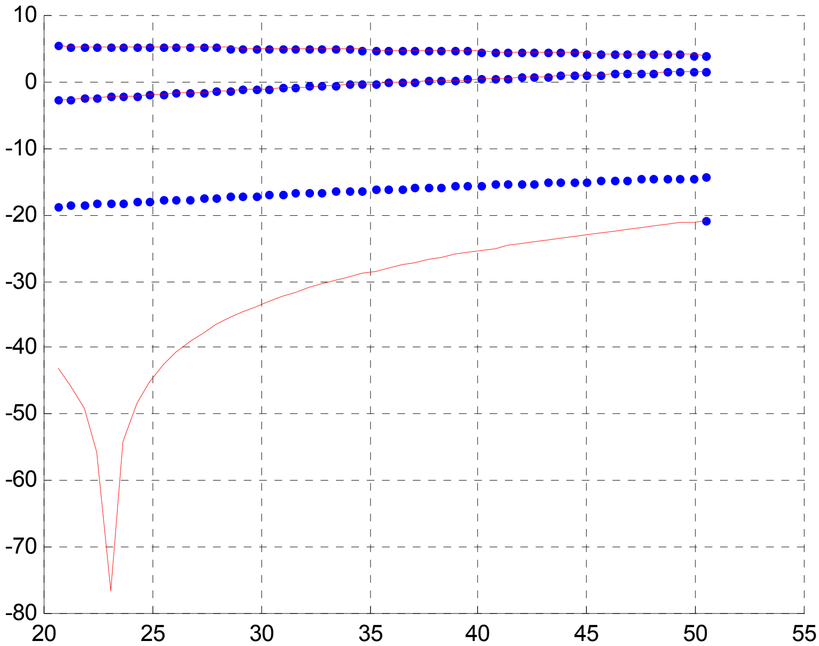

Figure 2: The same as figure 1, but at the equator (the conditioning is worse).

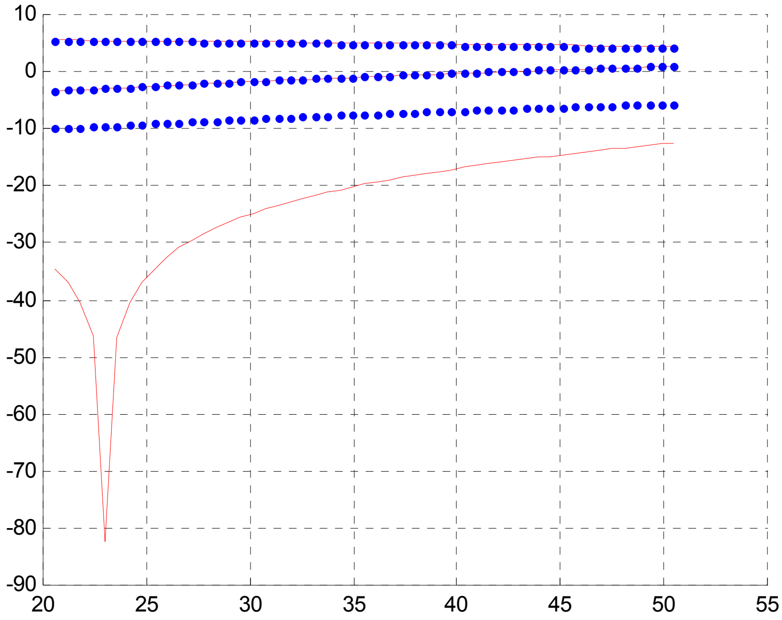

Figure 3: The same as figure 1, but at high latitudes (better conditioning).

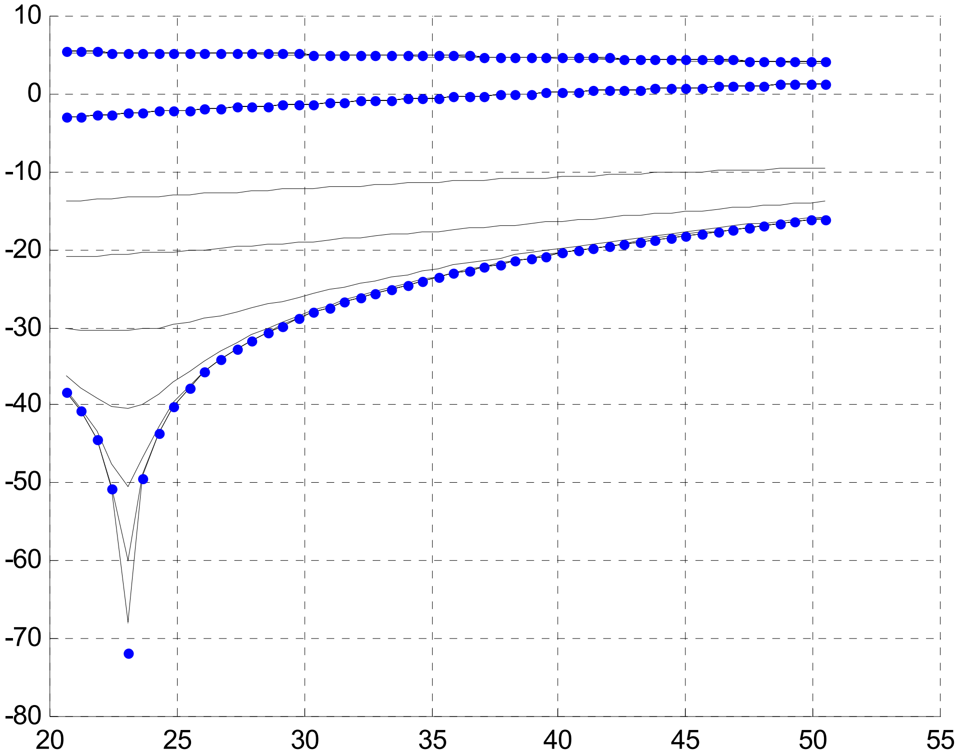

Figure 4: Effect of the decrease of the weight of the left acquisitions: the two upper black dotted lines correspond to up down and EW components. The seven lower continuous black lines correspond to the NS component, dedicating to left acquisitions half of the total (uppermost black continuous curve); then, the weight decreases of a factor 10 at each line (0.5, 0.5e-1, 0.5e-2, etc). The lowest dotted line corresponds to right acquisitions only. For instance, at incidence angle 25^0, dedicating to the left acquisitions only 5% of the total, we get –20dB as the SNR loss of the NS component.

Another possible estimate of the NS component

The phase dispersion of the LOS measure of each pixel is σ_φ, and it is related to the coherence γ as follows.

$$\gamma = e^{-\sigma_\varphi^{\,2}/2} \rightarrow \sigma_\varphi = \sqrt{-2\log\gamma}$$

$$\text{If}: \quad \gamma = 0.7 \rightarrow \sigma_\varphi = 0.77$$

$$\sigma_x = \frac{1}{\sqrt{4}}\frac{\lambda}{4\pi}\sigma_\varphi \approx 1.7mm$$

For each quadruple of measures, the dispersion of the retrieved motion σ_X is about 1.7mm, in the case of UD motion. To have similar precision for the EW (NS) component we need 10 (250) pixels, with 23^0 and 45^0 acquisitions; if we have 23^0 - 26^0 acquisitions, we need about 250 (2000). An incoherent analysis [1] based on the differences between two images has some interest. The estimate of the shift δ of a function x(t), spectrally white up to pulsation π/T, T being the azimuth sampling interval, using a time series MT pixels long, with coherence γ, is:

$$x(t+\delta) \approx x(t) + \delta\frac{dx}{dt} + n(t) = x(t) + \delta y(t) + n(t)$$

$$y(t) = \frac{dx}{dt}$$

$$\hat{\delta} = \frac{E[x(t)y(t)]}{E[y^2(t)]}$$

$$\sigma_y^{\,2} = \frac{\pi^2\sigma_x^{\,2}}{3T^2}$$

$$\text{If} \quad \gamma = \frac{\sigma_x^{\,2}}{\sigma_x^{\,2} + \sigma_n^{\,2}}$$

$$\text{If} \quad \delta = 0:$$

$$2M \times E\left[\left(\frac{\hat{\delta}}{T}\right)^2\right] = \frac{\sigma_x^{\,2}\sigma_y^{\,2}}{T^2\sigma_y^{\,4}} = \frac{\sigma_x^{\,2}}{T^2\sigma_y^{\,2}} = \frac{1-\gamma}{\gamma}\frac{3}{\pi^2}$$

For γ=0.7, if we wish to have a dispersion of about 1.5mm, i.e. $\delta=3\text{x}10^{-4}\text{xT}$, we need a number of pixels M=400000. In order to have a good quality NS component, we need thousands of points characterized by high coherence. In semi rural areas, where we expect say 50 PS/km^2, we need at least 50 km^2 with the same motion to be able to make a very good estimate of NS component (2mm).However, this area would correspond to a grid with 7km side, still much finer than that usually achieved using GPS receivers. Thus, DIFSAR could improve on GPS even for its worst conditioned NS component. The image difference approach is more pixel hungry by two orders of magnitude. Moreover, it cannot be carried out, unless we have contiguous high coherence pixels, i.e. only in desert areas. However, if carried out on the moduli (3dB loss in precision or duplication of the pixels needed) it is independent of atmospheric noise.

Conclusion

DIFSAR can retrieve the 3 motion components, albeit with different quality. The UD and EW component quality does not change if the acquisitions are left, right, or mixed. The NS component quality improves even with a few left acquisitions; nonetheless, multiple incidence angles acquisitions yield reasonable quality NS components. Image differences techniques could be useful for low quality measurements of the NS component. They can be used only if the NS motion stays the same over vast, coherent areas.

References

[1] Cafforio, C.; Rocca, F.; Methods for measuring small displacements of television images, Information Theory, IEEE Transactions on , Volume: 22 Issue: 5 , Sep 1976 , Page(s): 573 –579.